Handout 06 Introduction to Numerical Geometry

1. Basic Concepts

(1) To be a valid metric, a function should be of the following properties, non-negativity, indiscernability, symmetry, and triangle inequality.

(2) Euclidean ball \[ \|x - x_0\|_2 = \sqrt{\sum_k \left| x^k - x_0^k \right|^2} \leq r \]; \[ l_1 \)-ball \[ \|x - x_0\|_1 = \sqrt{\sum_k \left| x^k - x_0^k \right|} \leq r \]; \[ l_\infty \)-ball \[ \|x - x_0\|_\infty = \max_k \left| x^k - x_0^k \right| \leq r \]

(3) A bijective (one-to-one and onto) continuous function with a continuous inverse is called a homeomorphism.

(4) Two metric spaces \((X, d)\) and \((Y, \delta)\) are equivalent if there exists a distance-preserving map (isometry) \( \varphi : (X, d) \to (Y, \delta) \) satisfying \( \delta \circ (\varphi \times \varphi) = d \); such \((X, d)\) and \((Y, \delta)\) are called isometric.

(5) Fast marching is an efficient algorithm to compute the geodesic distance between a pair of points on a mesh.

(6) Sampling density and sampling efficiency are two metrics to quantify the sampling quality; farthest point sampling is a greedy algorithm to achieve a \( r \)-covering and \( r \)-separated sampling of the set \( X \).

(7) By moving the origin to the mass center, the translation isometry can be resolved; then, three axes can be obtained by using PCA (or K-L) transforms.

(8) Iterative closest point (ICP) is a classical algorithm to register or to compare a pair of point sets up to similarity transformations.

2. Matlab Programming

(1) Study the demo program “FastMarching” provided on our course website.

(2) Study the demo program “EuclideanIsometryRemoval”.

(3) Study the demo program “ICP”.