Handout 06 Geometric Transformations and Image Registration

1. Basic Concepts

(1) Geometric transformations can be used to create thumbnail views, change digital video resolution, correct distortions caused by viewing geometry, and align multiple images of the same scene.

(2) Geometric transformations can be classified as isometry transformation, similarity transformation, affine transformation, and projective transformation.

(3) Isometry transformation can be expressed by the formula \( x' = \begin{bmatrix} R & t \\ 0^T & 1 \end{bmatrix} x \), where \( R \) is a 2 by 2 orthogonal matrix and \( t \) is a 2 by 1 vector. \( R \) is an orthogonal matrix; Euclidean distance is preserved; it has three degrees of freedom, two for translation, and one for rotation.

(4) Similarity transformation can be expressed by the formula \( x' = \begin{bmatrix} sR & t \\ 0^T & 1 \end{bmatrix} x \), where \( R \) is a 2 by 2 orthogonal matrix and \( t \) is a 2 by 1 vector. \( R \) is an orthogonal matrix; Similarity ratio (the ratio of two lengths) is preserved; it has four degrees of freedom, two for translation, one for rotation, and one for scaling.

(5) Affine transformation can be expressed by the formula \( x' = \begin{bmatrix} A & t \\ 0^T & 1 \end{bmatrix} x \), where \( A \) is a 2 by 2 nonsingular matrix and \( t \) is a 2 by 1 vector. \( A \) is a non-singular matrix. Ratio of lengths of parallel line segments is preserved; it has six degrees of freedom, two for translation, one for rotation, one for scaling, one for scaling direction, and one for scaling ratio.

(6) Projective transformation can be expressed by the formula \( \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \), where \( c \) is a constant. Cross ratio preserved; Though it has 9 parameters, it has 8 degrees of freedom, since only the ratio is important in the homogeneous coordinates.

(7) The basic procedures to apply \( T \) to \( f \) to get transformed \( g \) includes, 1> evaluate \( (w_k, z_k) = T^{-1} [(x_k, y_k)] \); 2> evaluate \( f (w_k, z_k) \); 3> \( g (x_k, y_k) = f (w_k, z_k) \). Interpolation is usually required in such a process since the images are discrete.

(8) A typical image registration algorithm may include the following steps, detecting features, matching corresponding features, inferring geometric transformation, and use the geometric transformation to align one image with the other.
2. Related Matlab routines

maketform, imtransform, cpselect, cp2tform

3. Matlab Programming

(1) Run the demo “Display images in the same coordinates”
(2) Run the demo “Manual registration”
(3) Take two pictures sharing something in common. Try to register one of them to the other using the “manual registration” approach. Do you have any idea on how to stitch the two images together?