Lecture 5
Image Restoration

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Contents

• What is image restoration?
• A model of the image degradation/restoration process
• Noise models
• Additive random noise reduction
• Periodic noise reduction
What is Image Restoration?

Image restoration attempts to restore images that have been degraded

- Identify the degradation process and attempt to reverse it
- Similar to image enhancement, but more objective
What is Image Restoration?

• Image enhancement VS image restoration
  • Image enhancement is largely a subjective process; enhancement techniques basically are heuristic procedures designed to manipulate an image in order to take advantage of the psychophysical aspects of the human visual system
  • Image restoration is an objective process
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A Model of the Image Degradation/Restoration Process

• Degradation process modeling
  • A degradation function with an **additive noise** term

\[ f(x, y) \rightarrow \text{Degradation function } H \rightarrow g(x, y) \rightarrow \text{Restoration filters} \rightarrow f^*(x, y) \]

\[ \eta(x, y) \]
A Model of the Image Degradation/Restoration Process

• Degradation process modeling
  • A degradation function with an \textit{additive noise} term

If $H$ is a linear, position-invariant process, the degraded image is given in the spatial domain by

$$g(x, y) = h(x, y) * f(x, y) + \eta(x, y)$$

where “*” denotes the convolution

The equivalent frequency domain representation is

$$G(u, v) = H(u, v)F(u, v) + N(u, v)$$
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Noise Models

The sources of noise in digital images arise during image acquisition (digitization) and transmission

- Imaging sensors can be affected by ambient conditions
- Interference can be added to an image during transmission
Noise Models

We can consider a noisy image to be modelled as follows:

\[ g(x, y) = f(x, y) + \eta(x, y) \]

where \( f(x, y) \) is the original image pixel, \( \eta(x, y) \) is the noise term and \( g(x, y) \) is the resulting noisy pixel.

If we can estimate the model of the noise in an image, it will help us to figure out how to restore the image.
Noise Models

There are many different models for the image noise term $\eta(x, y)$:

- Gaussian
  - Most common model
- Rayleigh
- Erlang (Gamma)
- Exponential
- Uniform
- Impulse
  - *Salt and pepper* noise
Noise Example

The test pattern to the right is ideal for demonstrating the addition of noise.

The following slides will show the result of adding noise based on various models to this image.
Noise Example (cont...)

Gaussian  Rayleigh  Gamma
Noise Example (cont...)

Exponential  Uniform  Impulse
Periodic Noise

• Periodic noise typically arises from electrical or electromechanical interference during image acquisition

• It is a spatially dependent noise
Periodic Noise

- Periodic noise can be modeled as sinusoid waves

In spatial domain,

\[ r(x, y) = A \sin \left[ \frac{2\pi u_0 (x + B_x)}{M} + \frac{2\pi v_0 (y + B_y)}{N} \right] \]

where \( A \) is the amplitude, \( u_0 \) and \( v_0 \) determine the sinusoidal frequencies, \( B_x \) and \( B_y \) are phase displacements with respect to the origin.
Periodic Noise

- Periodic noise can be modeled as sinusoid waves

In frequency domain,

\[ R(u, v) = \frac{j^{AMN}}{2} \left[ e^{-j2\pi(u_0 B_x / M + v_0 B_y / N)} \delta(u+u_0, v+v_0) - e^{j2\pi(u_0 B_x / M + v_0 B_y / N)} \delta(u-u_0, v-v_0) \right] \]

Two symmetric spikes in the Fourier domain

Sinusoidal wave in the spatial domain
Periodic Noise

- Periodic noise can be modeled as sinusoid waves

Another example

Source code for this demo is available on our course website
Periodic Noise

• Periodic noise can be modeled as sinusoid waves

Image with periodic noises

Original image  Periodic noise pattern
Periodic Noise

• Periodic noise can be modeled as sinusoid waves

Image with periodic noises

Image with periodic noise

Fourier spectrum
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The Degradation Model Can be Simplified

General image degradation model

\[ g(x, y) = h(x, y) * f(x, y) + \eta(x, y) \]

When only noise is considered, the above model can be simplified to

\[ g(x, y) = f(x, y) + \eta(x, y) \]

- When the noise is additive random, spatial filtering methods can be used
- When the noise is periodic, frequency filtering can be used
Additive Random Noise Reduction

• Arithmetic mean filter
  • It computes the average value of the corrupted image $g(x, y)$ in the area defined by a neighborhood window $S_{xy}$
  • Blurs the image to remove noise
  • It can be computed as follows

$$\hat{f}(x, y) = \frac{1}{mn} \sum_{(s, t) \in S_{xy}} g(s, t)$$

($m$ and $n$ are the height and width of $S_{xy}$)

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The 3 by 3 arithmetic mean filter
Additive Random Noise Reduction

- Geometric mean filter
  - A geometric mean filter achieves smoothing comparable to the arithmetic mean filter, but it tends to lose less image detail in the process
  - It can be computed as follows

\[
\hat{f}(x, y) = \left[ \prod_{(s, t) \in S_{xy}} g(s, t) \right]^{1/mn}
\]
Additive Random Noise Reduction

• Harmonic mean filter
  • It works well for salt noise, but fails for pepper noise
  • It also works well for Gaussian noise
  • It can be computed as follows

\[ \hat{f}(x, y) = \frac{mn}{\sum_{(s,t) \in S_{xy}} \frac{1}{g(s,t)}} \]
Additive Random Noise Reduction

- Contraharmonic mean filter
  - It is well suited for reducing salt-and-pepper noise
  - It can be computed as follows
    \[
    \hat{f}(x, y) = \frac{\sum_{(s,t) \in S_{xy}} g(s, t)^Q \cdot g(s, t)^{Q+1}}{\sum_{(s,t) \in S_{xy}} g(s, t)^Q}
    \]
    where \(Q\) is called the order of the filter
  - If \(Q>0\), the filter eliminates pepper noise; if \(Q<0\), it eliminates salt noise; it cannot do both simultaneously
Additive Random Noise Reduction—Examples

Original image

Image corrupted by Gaussian Noise

After a 3*3 arithmetic mean filter

After a 3*3 geometric mean filter
Additive Random Noise Reduction—Examples

Original image

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Additive Random Noise Reduction—Examples

with pepper noise

with salt noise

contraharmonic filtering with $Q = 1.5$

contraharmonic filtering with $Q = -1.5$

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Additive Random Noise Reduction—Examples

Choosing the wrong value for $Q$ when using the contraharmonic filter can have drastic results

![Image showing examples of noise reduction with and without the correct value of $Q$.]
Additive Random Noise Reduction

• Order Statistics Filters
  • Spatial filters whose response is based on ordering (ranking) the values of the pixels contained in the image area encompassed by the filter
  • Commonly used order statistics filters include
    • Median filter
    • Max and min filter
    • Midpoint filter
    • Alpha trimmed mean filter
Additive Random Noise Reduction

- Median filter
  - For certain types of random noise, it provides excellent noise-reduction capabilities, with considerably less blurring than linear smoothing filters of the same size
  - Particularly good when salt and pepper noise is present
  - It replaces the value of a pixel by the median of the intensity levels in the neighborhood of that pixel:

\[
\hat{f}(x, y) = \text{median}\{g(s, t)\}_{(s, t) \in S_{xy}}
\]
Additive Random Noise Reduction

• Max filter and Min filter
  • Max filter is good for pepper noise
  • Min filter is good for salt noise

Max filter

\[
\hat{f}(x, y) = \max_{(s, t) \in S_{xy}} \{g(s, t)\}
\]

Min filter

\[
\hat{f}'(x, y) = \min_{(s, t) \in S_{xy}} \{g(s, t)\}
\]
Additive Random Noise Reduction

- **Midpoint filter**
  - It simply computes the midpoint between the maximum and minimum values in the area encompassed by the filter.
  - Good for Gaussian noise and uniform noise.

\[
\hat{f}(x, y) = \frac{1}{2} \left[ \max_{(s,t) \in S_{xy}} \{ g(s, t) \} + \min_{(s,t) \in S_{xy}} \{ g(s, t) \} \right]
\]
Additive Random Noise Reduction

• Alpha-Trimmed Mean Filter:
  • It is useful in situations involving multiple types of noise, such as a combination of salt-and-pepper noise and Gaussian noise

Suppose that we delete the $d/2$ lowest and $d/2$ highest intensity values of $g(s, t)$ in the neighborhood of $S_{xy}$. Alpha-trimmed mean filter is defined as

$$
\hat{f}(x, y) = \frac{1}{mn-d} \sum_{(s,t) \in S_{xy}} g_r(s,t)
$$

where $g_r(s, t)$ represents the remaining $mn-d$ pixels
Additive Random Noise Reduction—Examples

Original image
Additive Random Noise Reduction—Examples

- with pepper noise
  - filtered by max filter
- with salt noise
  - filtered by min filter
Additive Random Noise Reduction—Examples

- Image corrupted by uniform noise
- Filtered by 5*5 arithmetic mean filter
- Filtered by 5*5 median filter
- Image further corrupted by salt and pepper noise
- Filtered by 5*5 geometric mean filter
- Filtered by 5*5 alpha-trimmed mean filter

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Additive Random Noise Reduction

• Adaptive filters
  • The filters discussed so far are applied to an entire image without any regard for how image characteristics vary from one point to another
  • The behavior of adaptive filters changes depending on the characteristics of the image inside the filter region
  • We will take a look at the adaptive median filter
Additive Random Noise Reduction

• Adaptive median filter
  • The median filter performs relatively well on impulse noise as long as the spatial density of the impulse noise is not large
  • The adaptive median filter can handle much more spatially dense impulse noise, and also performs some smoothing for non-impulse noise
  • The key insight in the adaptive median filter is that the filter size changes depending on the characteristics of the image
Additive Random Noise Reduction

- Adaptive median filter

Remember that filtering looks at each original pixel image in turn and generates a new filtered pixel

First examine the following notation:

- $z_{\text{min}}$ = minimum grey level in $S_{xy}$
- $z_{\text{max}}$ = maximum grey level in $S_{xy}$
- $z_{\text{med}}$ = median of grey levels in $S_{xy}$
- $z_{xy}$ = grey level at coordinates $(x, y)$
- $S_{\text{max}}$ = maximum allowed size of $S_{xy}$
Additive Random Noise Reduction

• Adaptive median filter

Stage A:  
\[ A_1 = z_{med} - z_{min} \]
\[ A_2 = z_{med} - z_{max} \]
If \( A_1 > 0 \) and \( A_2 < 0 \) //the median is not an impulse
   Go to stage B
Else //the median is an impulse
   increase the window size
If window size \( \leq S_{max} \)
   repeat stage A
Else output \( z_{med} \)

Stage B:  
\[ B_1 = z_{xy} - z_{min} \]
\[ B_2 = z_{xy} - z_{max} \]
If \( B_1 > 0 \) and \( B_2 < 0 \) //the central pixel is not an impulse
   output \( z_{xy} \)
Else output \( z_{med} \)
Additive Random Noise Reduction

• Adaptive median filter
  • The key to understanding the algorithm is to remember that the adaptive median filter has three purposes:
    • Remove impulse noise
    • Provide smoothing of other noise
    • Reduce distortion (less blurring effects)

Next page shows an example for comparing median filter and adaptive median filter
Additive Random Noise Reduction—Examples

- Original Image
- Image with salt and pepper noise
- Result by median filtering (11x11)
- Result by adaptive median filtering (max window size 11x11)
Additive Random Noise Reduction—Examples

Original image
Additive Random Noise Reduction—Examples

Image with salt and pepper noise
Additive Random Noise Reduction—Examples

Filtering result by a 11*11 median filter
Filtering result by an adaptive median filter, whose maximum window size is 11*11
Additive Random Noise Reduction—Examples

• Adaptive median filter

How to implement the example you have just seen? Assignment
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• Periodic noise reduction
  • Selective filtering
  • Periodic noise reduction by selective filters
Selective Filtering

- Review: low pass filters (lecture 4)

The transfer function for the ideal low pass filter can be given as:

\[ H(u, v) = \begin{cases} 
1 & \text{if } D(u, v) \leq D_0 \\
0 & \text{if } D(u, v) > D_0 
\end{cases} \]

where \( D(u, v) \) is the distance of \((u, v)\) to the frequency centre \((0, 0)\) and it is given as: \( D(u, v) = \sqrt{u^2 + v^2} \)
Selective Filtering

- Bandreject filters (remove frequency components within a specific range)

**Ideal**

\[
H(u, v) = \begin{cases} 
0 & \text{if } D_0 - \frac{W}{2} \leq D(u, v) \leq D_0 + \frac{W}{2} \\
1 & \text{otherwise}
\end{cases}
\]

**Butterworth**

\[
H(u, v) = \frac{1}{1 + \left[ \frac{WD(u, v)}{D^2(u, v) - D_0^2} \right]^{2n}}
\]

**Gaussian**

\[
H(u, v) = 1 - e^{-\left[ \frac{D^2(u, v) - D_0^2}{WD(u, v)} \right]^2}
\]

where \( W \) is the width of the band
Selective Filtering

- Bandreject filters (*remove frequency components within a specific range*)

ideal band reject filter
Selective Filtering

- Bandreject filters (remove frequency components within a specific range)

Butterworth band reject filter (of order 1)
Selective Filtering

- Bandreject filters (*remove frequency components within a specific range*)

Gaussian band reject filter
Selective Filtering

• Bandpass filters
  • Let only a portion of the frequency components pass
  • They can be constructed by

\[ H_{BP}(u, v) = 1 - H_{BR}(u, v) \]
Selective Filtering

- Bandpass filters
  - Let only a portion of the frequency components pass

ideal band pass filter
Selective Filtering

- Bandpass filters
  - Let only a portion of the frequency components pass

Butterworth band pass filter (of order 1)
Selective Filtering

- Bandpass filters
  - Let only a portion of the frequency components pass

Gaussian band pass filter
Selective Filtering

• Notch filters
  • They are the most useful of the selective filters
  • A notch filter rejects (or passes) frequencies in a predefined neighborhood
  • Zero-phase-shifted filters must be symmetric about the origins, so a notch with center at \((u_0, v_0)\) must have a corresponding notch at location \((-u_0, -v_0)\)
Selective Filtering

- Notch filters
  - Notch reject filters are constructed as products of highpass filters whose centers are translated to the centers of the notches

\[
H_{NR}(u, v) = \prod_{k=1}^{Q} H_k(u, v)H_{-k}(u, v)
\]

where \(H_k(u, v), H_{-k}(u, v)\) are highpass filters whose centres are at \((u_k, v_k), (-u_k, -v_k)\), respectively

The distance computations for each filter are

\[
D_k(u, v) = (u - u_k)^2 + (v - v_k)^2
\]

\[
D_{-k}(u, v) = (u + u_k)^2 + (v + v_k)^2
\]
Selective Filtering

- Notch filters—An example

Butterworth notch reject filter of order $n$, containing three notch pairs,

$$H_{NR}(u, v) = \prod_{k=1}^{3} \left[ \frac{1}{1 + \left[ \frac{D_{k0}}{D_k(u, v)} \right]^{2n}} \right] \left[ \frac{1}{1 + \left[ \frac{D_{k0}}{D_{-k}(u, v)} \right]^{2n}} \right]$$

The constant $D_{k0}$ is the same for each pair of notches, but it can be different for different pairs.

A **notch pass filter** is obtained from a notch reject filter by

$$H_{NP}(u, v) = 1 - H_{NR}(u, v)$$
Selective Filtering

• Notch filters—An example

A Butterworth notch reject filter, containing one notch pair
Selective Filtering

- Notch filters—An example

A Butterworth notch reject filter, containing two notch pairs

Source code for this demo is available on our course website
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Periodic noise reduction by selective filters

Typically arises due to electrical or electromagnetic interference

Gives rise to regular noise patterns in an image

Frequency domain techniques in the Fourier domain are most effective at removing periodic noise
Periodic noise reduction by selective filters

- Notch filters can efficiently remove the periodic noise
  - Step 1: analyze the Fourier spectrum \( F \) of the image
  - Step 2: identify the locations of the peaks in \( F \)
  - Step 3: construct a notch reject filter \( H \) in Fourier domain, whose centers are at peaks
  - Step 4: use \( H \) to filter \( F \) to get the result
Periodic noise reduction by selective filters

- Notch filters can efficiently remove the periodic noise

Step 1: analyze the Fourier spectrum $F$ of the image

Noisy image  

Fourier spectrum $F$
Periodic noise reduction by selective filters

- Notch filters can efficiently remove the periodic noise

Step 2: identify the locations of the peaks in $F$

Padded image
Periodic noise reduction by selective filters

- Notch filters can efficiently remove the periodic noise

Step 2: identify the locations of the peaks in $F$
Periodic noise reduction by selective filters

• Notch filters can efficiently remove the periodic noise

Step 3: construct a notch reject filter $H$ in Fourier domain, whose centers are at peaks

Notch filter $H$
Periodic noise reduction by selective filters

- Notch filters can efficiently remove the periodic noise

Step 4: use $H$ to filter $F$ to get the result

$HF$ in the Fourier domain
Periodic noise reduction by selective filters

- Notch filters can efficiently remove the periodic noise

Step 4: use $H$ to filter $F$ to get the result

Final result in the spatial domain
Periodic noise reduction by selective filters

- Notch filters can efficiently remove the periodic noise.
  Let’s see the power of such a technology.

Noisy image

After de-noising
Thanks for your attention