

# Pattern Recognition, Fall 2012

## Assignment 1: Bayesian Decision Theory

**Due: November 8, 2012**

1. (30pts) A dishonest gambler has a box containing 10 dice which all look the same. However there are actually three types of dice.

- There are 6 dice of type A which are fair dice with  $\Pr(6 | A) = 1/6$  (where  $\Pr(6 | A)$  is the probability of getting a 6 in a throw of a type A die).
- There are 2 dice of type B which are biased with  $\Pr(6 | B) = 0.8$ .
- There are 2 dice of type C which are biased with  $\Pr(6 | C) = 0.04$ .

The gambler takes a die from the box at random and rolls it. Find the conditional probability that it is of type B given that it gives a 6.

2. Consider a three-class classification problem where the classes  $\omega_1, \omega_2, \omega_3$  are equally probable and have class-conditional probability distributions  $p(\mathbf{x}|\omega_i) = N(\boldsymbol{\mu}_i, \boldsymbol{\Sigma}_i)$  with

$$\boldsymbol{\mu}_1 = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \quad \boldsymbol{\mu}_2 = \begin{pmatrix} 2 \\ -2 \end{pmatrix} \quad \boldsymbol{\mu}_3 = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$$

and identical covariance matrices  $\boldsymbol{\Sigma}_1 = \boldsymbol{\Sigma}_2 = \boldsymbol{\Sigma}_3 = \sigma^2 \mathbf{I}$  where  $\sigma^2 = 0.25$ .

- (10 pts) Find the discriminant functions corresponding to the minimum-error decision rule.
- (10 pts) Find the decision regions and boundaries associated with this rule.
- (10 pts) Generate 100 random patterns from each of the three class-conditional distributions and plot them in the two-dimensional feature space. Sketch the decision boundaries of part (b) on this plot. (**Hint:** In Matlab, you can use the `mvnrnd` function for generating the random patterns, and `plot` and `ezplot` functions for plotting.)
- (10 pts) Find the maximum likelihood estimates of  $\boldsymbol{\mu}_i$  and  $\boldsymbol{\Sigma}_i, i = 1,2,3$ , using the sample patterns.
- (15 pts) Substitute these estimates to the Bayesian decision rule and find the decision boundaries. Sketch them along with the plots in part (c).
- (15 pts) Generate a new set of random patterns from each class, compute the maximum likelihood estimates of the parameters, and find the resulting decision boundaries. Prepare a new plot, like the one in part (e), that overlays the new decision boundaries on the new sample patterns. Compare these results to the previous case. Are the decision boundaries obtained using different samples different? Why / why not? Does this match your expectations?